Localisation of Dirac modes in a finite temperature SU(2)-Higgs model

György Baranka

Eötvös Loránd University
Budapest

In collaboration with Matteo Giordano
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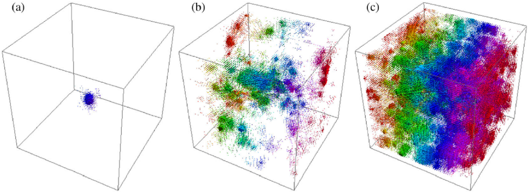
Introduction I

- The connection between deconfinement and chiral symmetry restoration at the finite temperature QCD transition is still not fully understood.
- Low Dirac modes could be key in understanding this connection.
- Chiral symmetry breaking is controlled by the density $\rho(\lambda)$ of low modes according to the Banks-Casher relation

$$\left|\langle\bar{\psi}(x)\psi(x)\rangle\right| \overset{m\to 0}{=} \pi\rho(0).$$

- Localisation of low modes of the Dirac operator was observed in QCD and other gauge theories above the deconfinement transition [Giordano and Kovács, 2021]



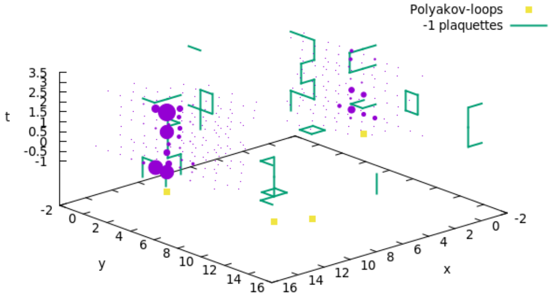
From Ref. [Ujfalusi et al., 2015]

- "Sea/islands" picture of localization: in the deconfined phase
  1. modes get "trapped" on "islands" of Polyakov-loop fluctuations [Bruckmann et al., 2011]
  2. gauge field fluctuations that decrease correlations in the temporal direction [Baranka and Giordano, 2022]

  within the "sea" of ordered Polyakov loops.
- Only ordering of the Polyakov-loop is needed $\rightarrow$ localization of low modes expected in a generic gauge theory

Introduction III

Example for localization in $\mathbb{Z}_2$ gauge theory
[Baranka and Giordano, 2021]



Introduction IV

- To test the "sea/islands" picture: check thermal transitions in which the Polyakov-loop gets ordered other than the deconfinement transition
- Another test of the "sea/islands" picture: changing the type of dynamic matter
- The fixed length SU(2)-Higgs model carries out both tests [Baranka and Giordano, 2023]

$$\lambda \to \infty$$

The model

$$S = -\frac{1}{2}\sum_n \left[\beta \sum_{\mu\nu} \mathrm{tr}\, U_{\mu\nu}(n) + \kappa \sum_{\mu} \mathrm{tr}\, G_\mu(n)\right]$$

$$U_{\mu\nu}(n) = U_\mu(n) U_\nu(n+\mu) U_\mu(n+\nu)^\dagger U_\nu(n)^\dagger$$

$$G_\mu(n) = \phi(n)^\dagger U_\mu(n) \phi(n+\mu)$$

$U_\mu(n)$ link variables (gauge field) and $\phi(n)$ site variables (scalar field).

$$\langle O \rangle = \frac{1}{Z}\int DUD\phi \exp(-S[U,\phi]) \cdot O[U,\phi]$$

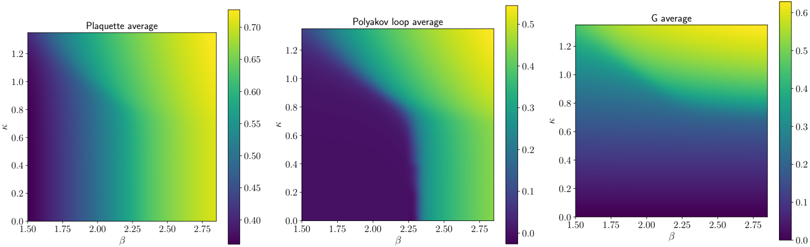
phase diagram was studied at zero temperature [Bonati et al., 2010] $\rightarrow$ do it at finite temperature $\rightarrow$ N_t fixed, $T = \frac{1}{N_t a}$

Gauge observables

- $U(n) = \frac{1}{24}\sum_{\mu<\nu} \mathrm{tr}\, U_{\mu\nu}(n) \rightarrow$ Plaquettes
- $G(n) = \frac{1}{8}\sum_{\mu<\nu} \mathrm{tr}\, G_{\mu\nu}(n) \rightarrow$ to measure the Higgs field
  $\rightarrow$ trace of links with a proper gauge transformation
- $P(\vec{x}) = \mathrm{tr} \prod_{t=0}^{N_t-1} U_4(\vec{x}, t)$
- $\chi_O \propto \langle O^2 \rangle - \langle O \rangle^2$

Mapping the phase diagram

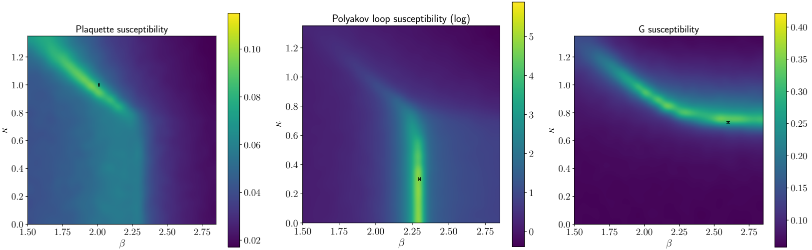
Our results at finite temperature:



As we expected from the study at $T = 0$.

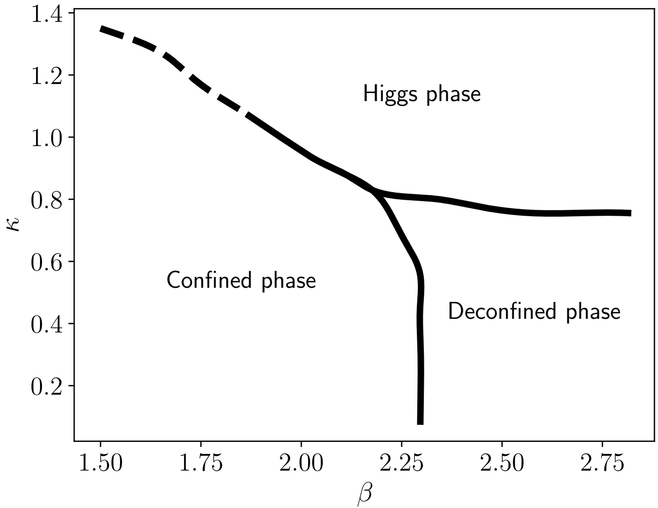
Mapping the phase diagram

Our results at finite temperature:

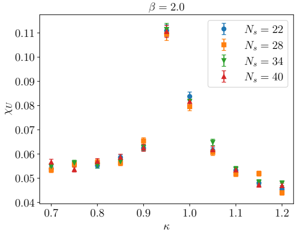


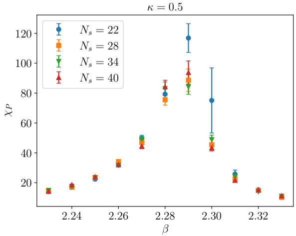
As we expected from the study at $T = 0$.

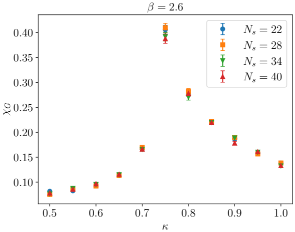
Sketch of phase diagram



Nature of transitions → crossover







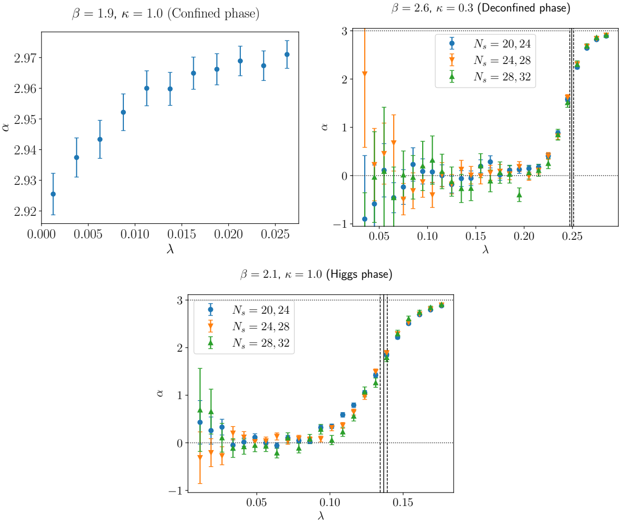
Participation ratio and fractal dimension

- $\mathrm{PR} = \frac{\text{occupied volume}}{\text{lattice volume}}$

$$\mathrm{PR}_l = \frac{1}{N_t N_s^3} \mathrm{IPR}_l^{-1}\,, \quad \mathrm{IPR}_l = \sum_n \|\psi_l(n)\|^4\,,$$

- $\mathrm{PR} \cdot V \sim N_s^\alpha$ at large N_s, α the fractal dimension of modes
- $\alpha = 0 \rightarrow$ Localized mode
- $\alpha = 3 \rightarrow$ Delocalizad mode

Fractal dimension



Spectral statistics

- Concentrate on the universal spectral statistical properties → unfold the spectrum

$$s = (\lambda_{i+1} - \lambda_i)\rho(\lambda_i)$$

- Delocalized modes → strongly correlated under small changes of the gauge field→ RMT (Random Matrix theory) statistics

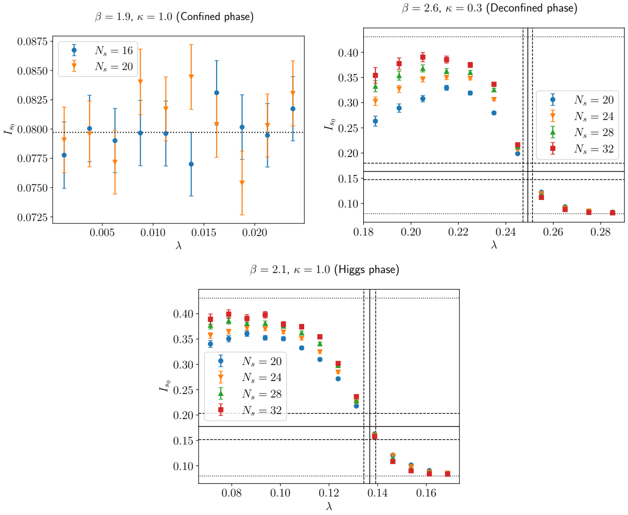
$$p(s) = a_\beta s^\beta e^{-b_\beta s^2}$$

- Localized modes → uncorrelated under small changes of the gauge field for large volumes → Poisson statistics

$$p(s) = e^{-s}$$

- $I_{s_0} = \int_0^{s_0} p(s)\mathrm{d}s$

Spectral statistics



Fractals around the mobility edge I

Generalized IPR:

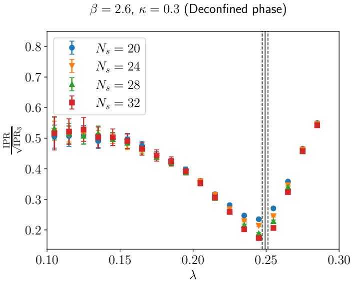
$$\mathrm{IPR}_q = \sum_x ||\psi(x)||^{2q}, \quad q \geq 2$$

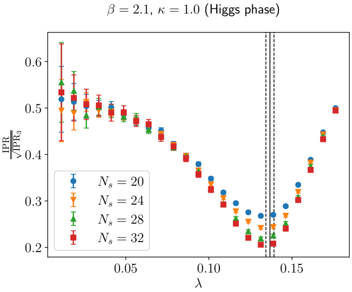
Generalized definition of fractal dimension:

$$\sqrt[q-1]{\mathrm{IPR}_q} \propto N_s^{-D_q}$$

Then the the ratio of the IPR for different q values

$$\frac{\sqrt[q_1-1]{\mathrm{IPR}_{q_1}}}{\sqrt[q_2-1]{\mathrm{IPR}_{q_2}}} \propto N_s^{D_{q_2}-D_{q_1}}$$


$\beta = 2.6$, $\kappa = 0.3$ (Deconfined phase)
$\frac{\mathrm{IPR}}{\sqrt{\mathrm{IPR}_3}}$
$N_s = 20$
$N_s = 24$
$N_s = 28$
$N_s = 32$
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.10
0.15
0.20
0.25
0.30
λ

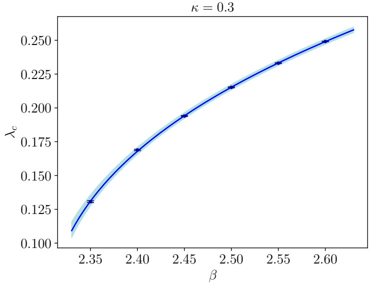

$\beta = 2.1$, $\kappa = 1.0$ (Higgs phase)
$\frac{\mathrm{IPR}}{\sqrt{\mathrm{IPR}_3}}$
$N_s = 20$
$N_s = 24$
$N_s = 28$
$N_s = 32$
0.6
0.5
0.4
0.3
0.2
0.05
0.10
0.15
λ

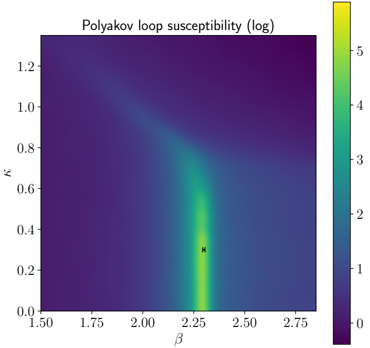
Disappearance of the mobility edge (Deconfined phase → confined phase)

Fitted function:

$$\lambda_c = a(\beta - \beta_c)^b$$

The mobility edge disappears in the transition region.





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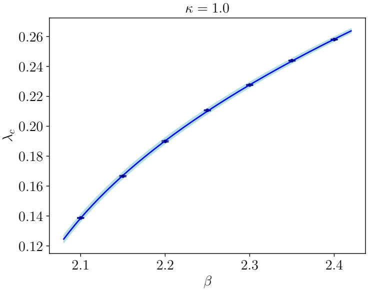
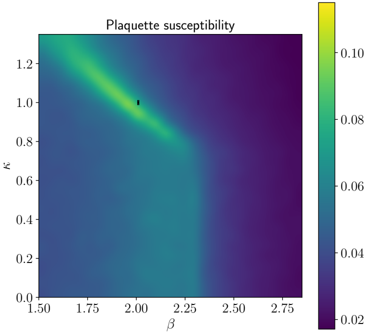
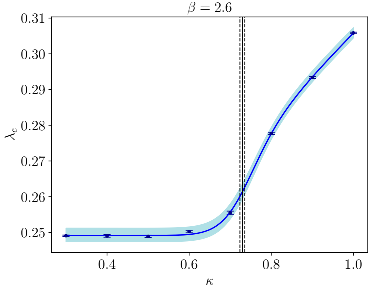


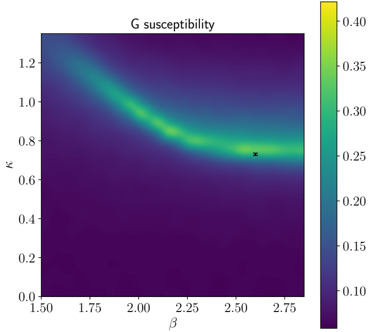


Fitted function:

$$\lambda_c = a \cdot (1 - \sigma(d \cdot (\kappa - \kappa_c))) + (b\kappa + c)\sigma(d \cdot (\kappa - \kappa_c))$$

$$\sigma(x) = \frac{1}{1 + e^{-x}}$$





Correlation with gauge observables

To measure the correlation with gauge observables define

$$\hat{\mathcal{O}}(\lambda, N_s) = \frac{\mathcal{O}(\lambda, N_s) - \langle O \rangle}{\delta O}$$

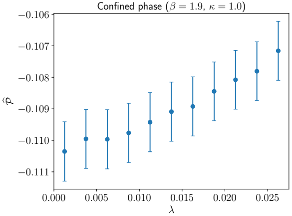
where

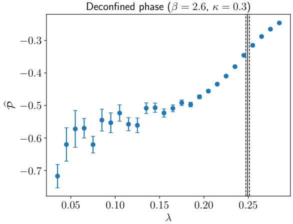
$$\mathcal{O}_l = \sum_n O(n) \cdot ||\psi(n)||_l^2$$

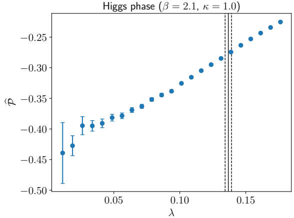
$$(\delta O)^2 = \langle U^2 \rangle - \langle U \rangle^2$$

Correlation with Polyakov loops

$$\hat{\mathcal{P}}(\lambda, N_s) = \frac{\mathcal{P}(\lambda, N_s) - \langle P \rangle}{\delta P}$$

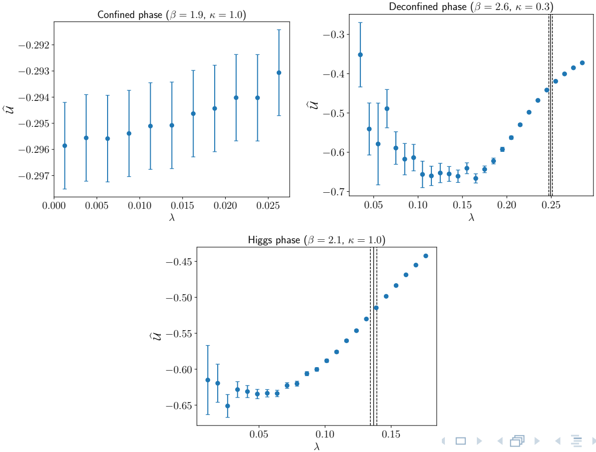






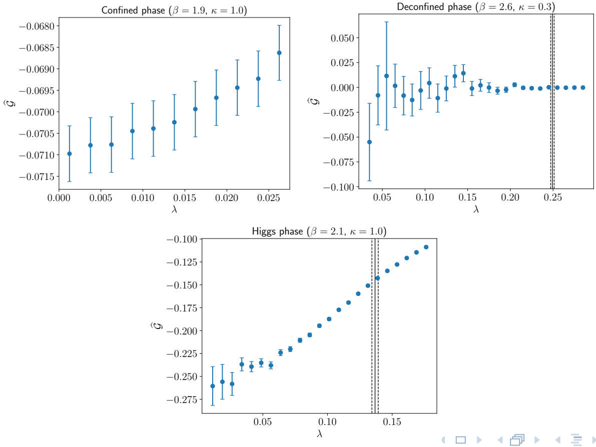
Correlation with plaquettes

$$\hat{\mathcal{U}}(\lambda, N_s) = \frac{\mathcal{U}(\lambda, N_s) - \langle U \rangle}{\delta U}$$



Correlation with the Higgs field

$$\hat{\mathcal{G}}(\lambda, N_s) = \frac{\mathcal{G}(\lambda, N_s) - \langle G \rangle}{\delta G}$$



Refined "sea/islands" picture

Unitary transformation: $D \to \Omega D \Omega^\dagger = H_{DA}$

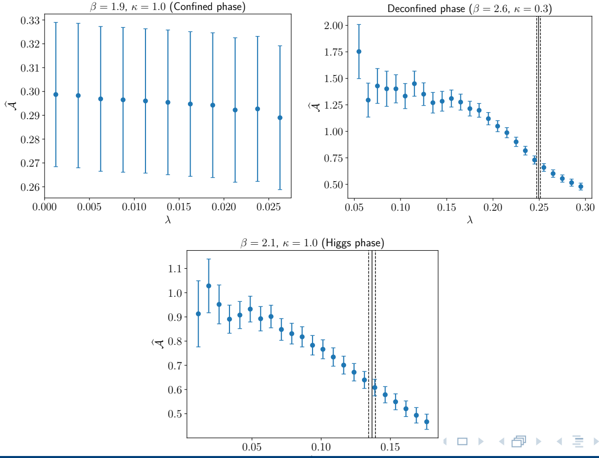
$$H_{DA} = \begin{bmatrix} E & 0 \\ 0 & -E \end{bmatrix} + \frac{1}{2i} \sum_{j=0}^{d} \eta_j \left(\begin{bmatrix} A_j & B_j \\ B_j & A_j \end{bmatrix} T_j - T_j^\dagger \begin{bmatrix} A_j^\dagger & B_j^\dagger \\ B_j^\dagger & A_j^\dagger \end{bmatrix} \right)$$

$$A(\vec{x}) = \sum_{j=\pm 1}^{\pm 3} ||A_j(\vec{x})||_F^2 \quad B(\vec{x}) = \sum_{j=\pm 1}^{\pm 3} ||B_j(\vec{x})||_F^2$$

- $A(\vec{x}) \gg B(\vec{(x)})$ $\;\to\;$ gauge field anti-correlated in the temporal direction
- $A(\vec{x}) \ll B(\vec{(x)})$ $\;\to\;$ gauge field correlated in the temporal direction

Correlation with A

$$\hat{\mathcal{A}}(\lambda, N_s) = \frac{\mathcal{A}(\lambda, N_s) - \langle A \rangle}{\delta A}$$



Conclusions

- Study of the relationship between localization and Polyakov-loop ordering extended: we studied the SU(2)-Higgs theory
- Phase diagram mapped at finite temperature (previously it was only studied at zero temperature [Bonati et al., 2010])
- "Sea/islands" picture is confirmed:
  1. works without regard to the dynamical matter
  2. and the type of the transition (the ordering of the Polyakov loop is enough)
- Possible extension:
  1. Studying the Laplacian operator (was also only studied at zero temperature [Greensite et al., 2005])
  2. or the low β, large κ part of the phase diagram (transition becomes weak: how the mobility edge disappears?)

References I

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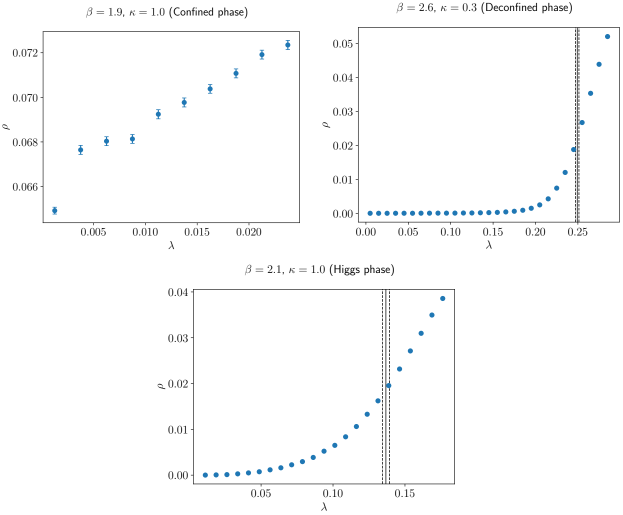
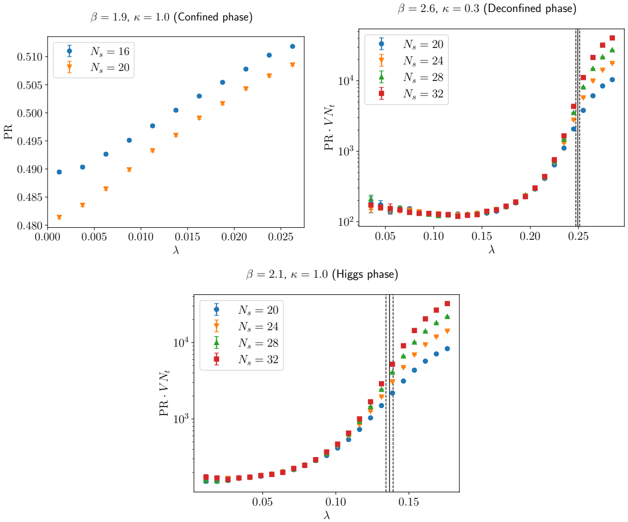
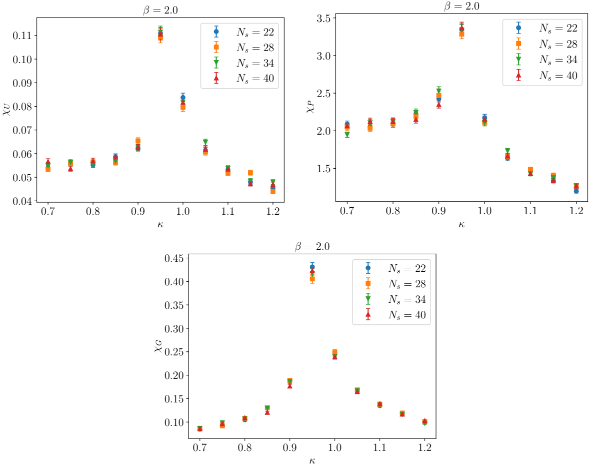
Spectral density



Participation ratio

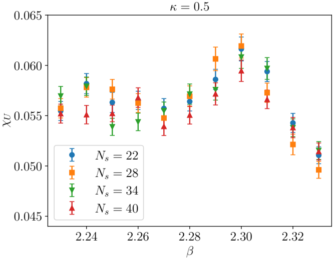


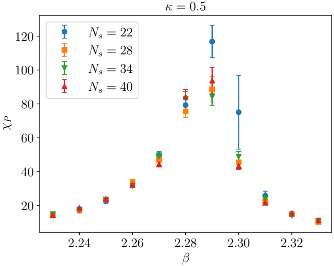
From confined to Higgs:



Nature of transitions → crossover

From confined to deconfined:





Nature of transitions → crossover

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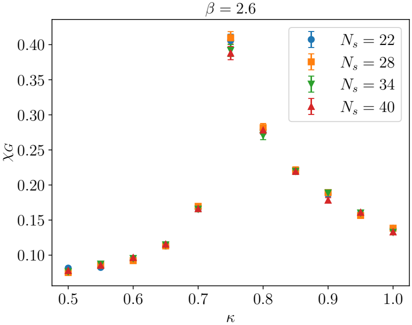

Mode size averaged up to $\frac{\lambda_c}{2}$